ProbLife: a Probabilistic Game of Life

BNAIC 2021

Simon Vandevelde, Joost Vennekens
KU Leuven – EAVISE – DTAI
13 September 2021

AI FLANDERS
BUILDING OUR DIGITAL FUTURE

KU LEUVEN eavise

Outline

1. Conway's Game of Life
2. GoL: extensions and variants
3. ProbLife
4. ProbLife in ProbLog
5. ProbLife in action

1 Conway's Game of Life

What is GoL

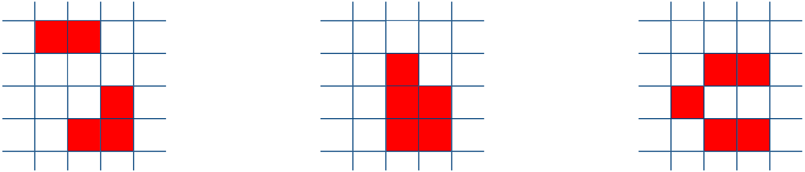
Game of Life

- *cellular automaton* invented by John Conway
- Grid of cells, some alive, some dead
- Next generation dictated by set of rules

What is GoL?

Rules of GoL

1. Cell survives if it has exactly 2 or 3 living neighbours
2. Cell is born if has exactly 3 living neighbours

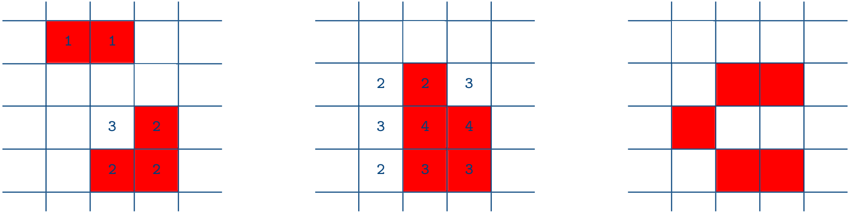


(a) Start state **(b)** State 2 **(c)** State 3

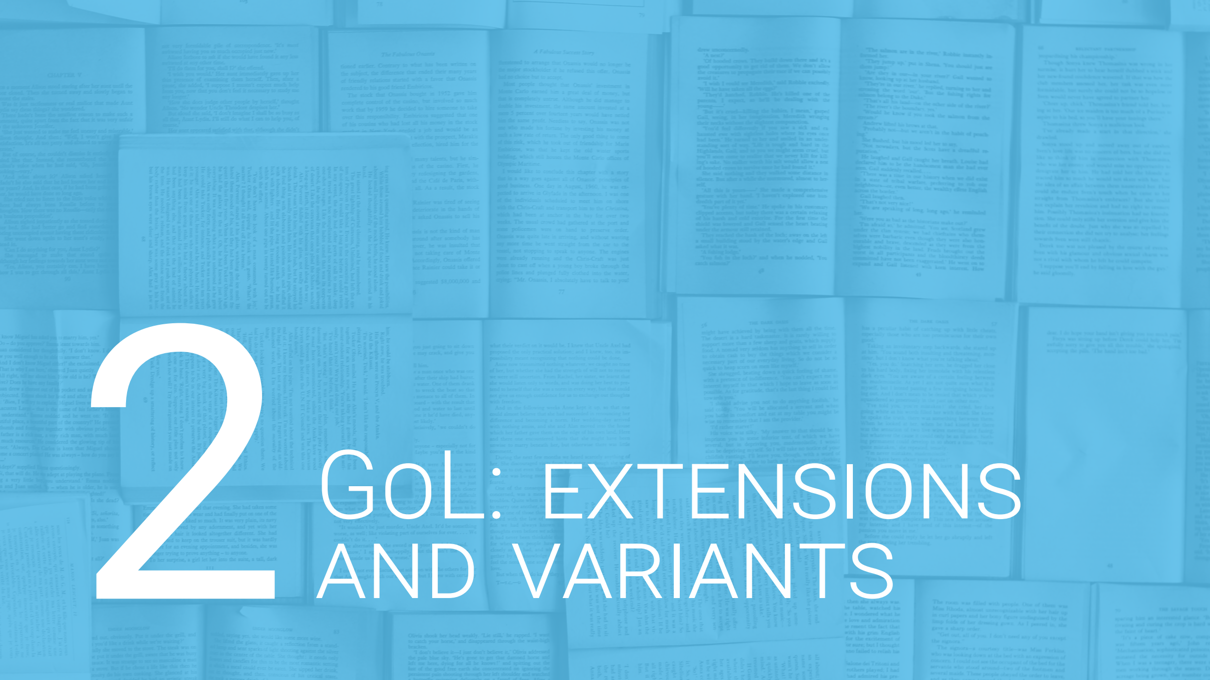
What is GoL?

Rules of GoL

1. Cell survives if it has exactly 2 or 3 living neighbours
2. Cell is born if has exactly 3 living neighbours



(a) Start state **(b)** State 2 **(c)** State 3



2 GoL: EXTENSIONS AND VARIANTS

Extensions & variants

Many extensions & variants exist!

- Alternate rulesets (*Flock*, *Day and Night*)
- Modified dimensionality (1D or 3D)
- Changing neighbourhood size (*Larger than Life*)
- Non-square grids
- **Probabilistic variants**
- **Continuous variants**
- . . .

Probabilistic Variants

PCAEGOL[1]

- Probability of misreading neighbour cell
- E.g. 20% of counting left neighbour incorrectly alive
- Errors are not consistent

Stochastic Game of Life (SGL)

- Survival rules have probability: not guaranteed!
- Extra probability for birth

[1] Aguilera-Venegas et al. 2019
[2] Monetti, Roberto A, and Ezequiel V Albano. 1997

Probabilistic Variants, cont.

GoL with temperature[1]

- Introduces temperature T
- Rules have probability, based on value of T
- T can also randomly flip cells, introducing chaos

Asynchronous Life[2]

- Chance not to update cell
- leads to asynchronicity

[1] Schulman L.S. and Seiden P.E. 1978
[2] Blok H.J. and Bergersen B, 1999

VARIANTS WITH CONTINUOUS ELEMENTS

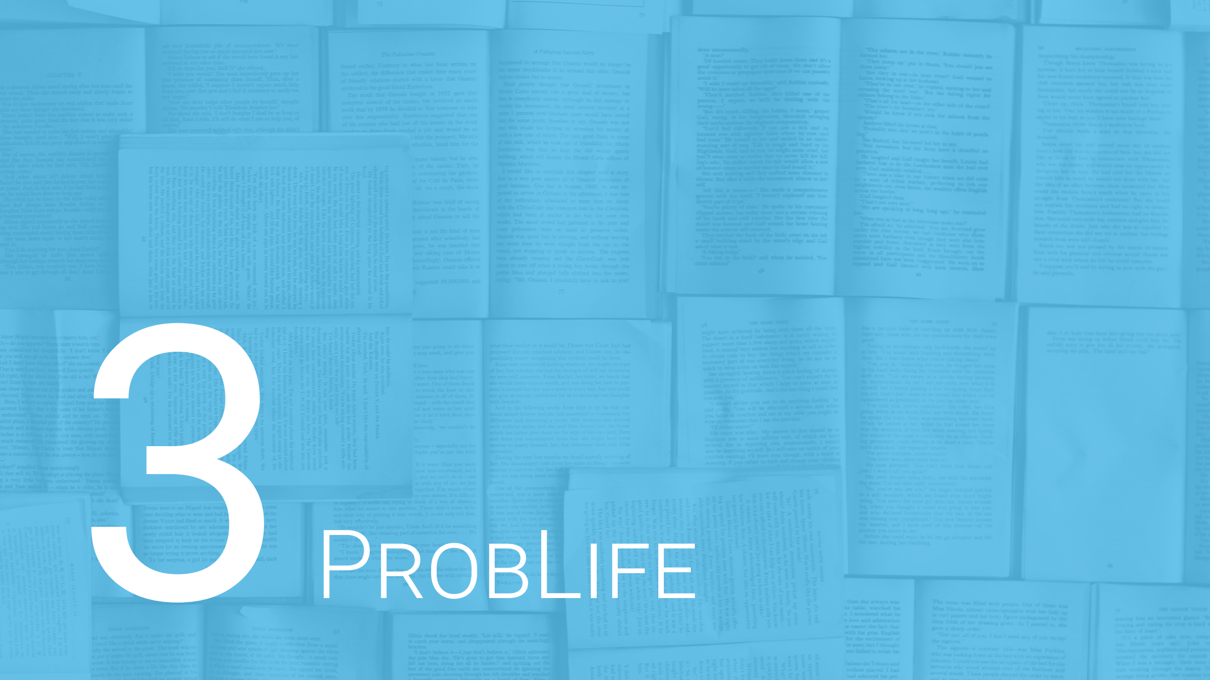
SmoothLife[1]

- Continuous grid
- Continuous neighbourhood

GoL at finite temperature[2]

- Continuous cell values in $[0..1]$
- When T rises, cell values become more “fuzzy”

Rafler, S. 2011
Adachi et al. 2004



3 ProbLife

What is ProbLife?

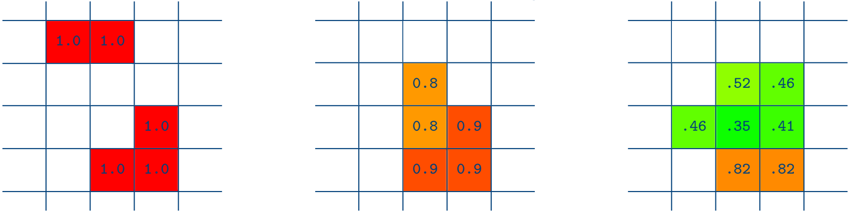
Probabilistic Game of Life:

- Cells can have any value in domain $[0..1]$
- Cell value represents probability of being alive
- Rules in ProbLife have probability

WHAT IS PROBLIFE?

Probabilistic Game of Life:

- Cells can have any value in domain $[0..1]$
- Cell value represents probability of being alive
- Rules in ProbLife have probability



(a) Start state **(b)** State 2 **(c)** State 3

Rules in ProbLife

ProbLife rule is of the form

$$p_c(n) = x \tag{1}$$

Rules in ProbLife

ProbLife rule is of the form

$$p_c(n) = x \tag{1}$$

with n number neighbours
c indicating type: s(urvive) or b(irth)
$x \in [0..1]$ the probability

Rules in ProbLife

ProbLife rule is of the form

$$p_c(n) = x \tag{1}$$

with n number neighbours
c indicating type: s(urvive) or b(irth)
$x \in [0..1]$ the probability

$$p_s(4) = 0.8 \tag{2}$$

Cells in ProbLife

Value of a cell at column i, row j, time $t+1$:

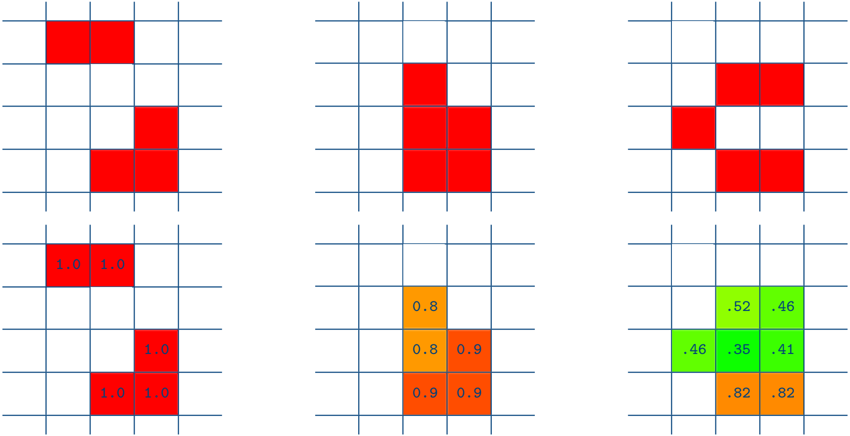
$$C_{t+1}(i,j) = \sum_{n=0}^{8} N_t(i,j,n) \times \Big(p_s(n) \times C_t(i,j) + p_b(n) \times \big(1 - C_t(i,j)\big)\Big). \quad (3)$$

Life in ProbLife

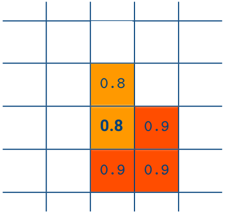
ProbLife generalizes the original:

$$\begin{aligned} p_b(3) &= 1. \\ p_s(2) &= 1. \\ p_s(3) &= 1. \\ p_s(i) &= 0, \text{ for all other combinations} \end{aligned} \tag{4}$$

Comparison

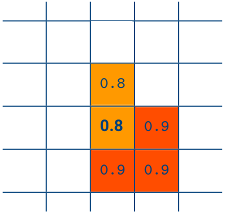


Rules in action



$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \qquad (5)$$
$$p_s(3) = 0.9$$

Rules in action

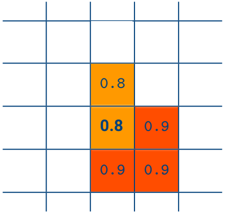


$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \qquad (5)$$
$$p_s(3) = 0.9$$

Probability of three living neighbours =

$$(0.8 \times 0.9 \times 0.9 \times 0.1) + (0.9 \times 0.9 \times 0.9 \times 0.2)$$
$$+(0.9 \times 0.9 \times 0.8 \times 0.1) + (0.9 \times 0.8 \times 0.9 \times 0.1)$$

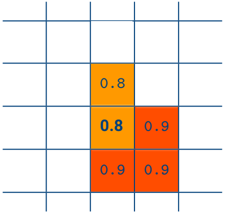
Rules in action



$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of three living neighbours = **0.340**

RULES IN ACTION

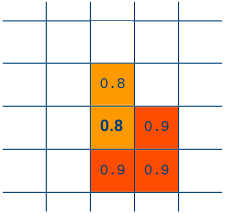


$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of survival with 3 neighbours:

$$p_s(3) \times 0.340 \times C_i \quad (6)$$

Rules in action

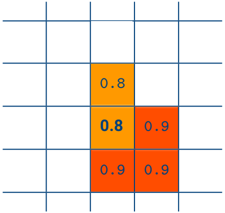


$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of survival with 3 neighbours:

$$0.9 \times 0.340 \times 0.8 = 0.245 \quad (6)$$

Rules in action

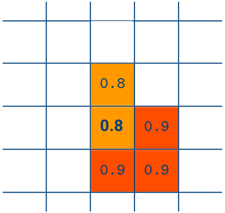


$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of birth with 3 neighbours:

$$p_b(3) \times 0.340 \times (1 - C_i) \quad (6)$$

Rules in action

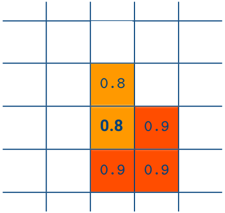


$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of birth with 3 neighbours:

$$0.8 \times 0.340 \times 0.2 = 0.054 \quad (6)$$

Rules in action

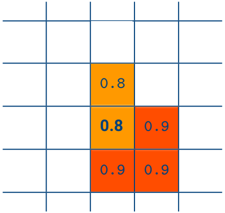


$$\begin{aligned} p_b(3) &= 0.8 \\ p_s(2) &= 0.9 \\ p_s(3) &= 0.9 \end{aligned} \quad (5)$$

Probability of 2 living neighbours:

$$\begin{aligned} &(0.8 \times 0.9 \times 0.1 \times 0.1) \times 3 \\ +&(0.9 \times 0.9 \times 0.2 \times 0.1) \times 3 = 0.07 \end{aligned} \quad (6)$$

Rules in action

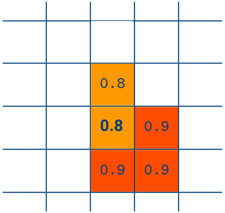


$$
\begin{aligned}
p_b(3) &= 0.8 \\
p_s(2) &= 0.9 \\
p_s(3) &= 0.9
\end{aligned} \tag{5}
$$

Probability of survival with 2 neighbours:

$$p_n(2) \times 0.07 \times C_i \tag{6}$$

Rules in action

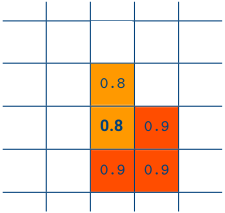


$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of survival with 2 neighbours:

$$0.9 \times 0.07 \times 0.8 = 0.0504 \quad (6)$$

Rules in action



$$p_b(3) = 0.8$$
$$p_s(2) = 0.9 \quad (5)$$
$$p_s(3) = 0.9$$

Probability of living cell at $t + 1$:

$$0.245 + 0.054 + 0.0504 = 0.35 \quad (6)$$

4 ProbLife in ProbLog

ProbLog: Probabilistic Logic Programming

ProbLog

- Probabilistic extension of Prolog
- A set of probabilistic facts and set of rules

$$P_r :: h \;\; :- \; b_1, \ldots, b_n \tag{7}$$

h: head

P_r: probability of evaluating head as *true*

b_i : body atoms

ProbLife rule in ProbLog

Survival rule $p_s(N) = z$ becomes

$$z :: alive(X, Y, T) \ :- T > 0, T_p \ is \ T - 1, alive(X, Y, T_p), \\ neigh(X, Y, T_p, N).$$

Birth rule $p_b(N) = z$ becomes

$$z :: alive(X, Y, T) \ :- T > 0, T_p \ is \ T - 1, not(alive(X, Y, T_p)), \\ neigh(X, Y, T_p, N).$$

ProbLife example

$$alive(X,Y,0) \ :- \ initAlive(X,Y).$$

$$0.9 :: alive(X,Y,T) \ :- T > 0, T_p \ is \ T-1, alive(X,Y,T_p), neigh(X,Y,T_p,3).$$

$$0.9 :: alive(X,Y,T) \ :- T > 0, T_p \ is \ T-1, alive(X,Y,T_p), neigh(X,Y,T_p,2).$$

$$0.8 :: alive(X,Y,T) \ :- T > 0, T_p \ is \ T-1, not(alive(X,Y,T_p)), neigh(X,Y,T_p,3).$$

5 ProbLife in action

Rule-based generative art

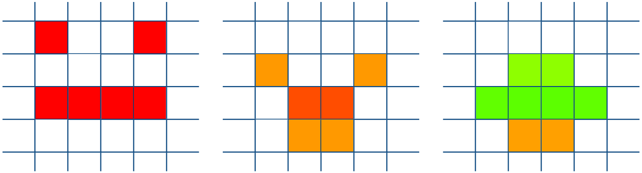


Figure: “Unamused tree”

Rule-based generative art

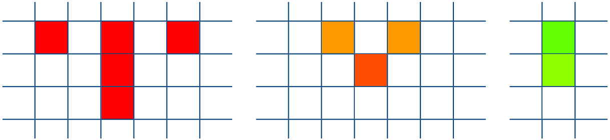


Figure: “Reverse Butterfly”, or, “Cold Water”

Rule-based generative art

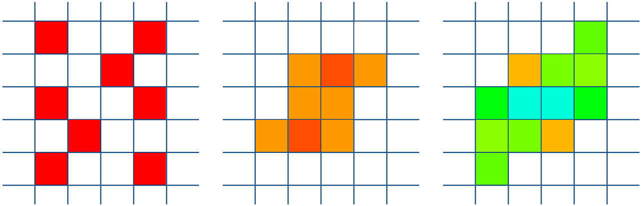


Figure: “Fata Morgana”

Thank you!

Thank you for your attention!
Questions?

s.vandevelde@kuleuven.be